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SOLIDIFICATION IN FLOW THROUGH CHANNELS AND INTO CAVITIES

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Abstract—The solutions of the Stefan problem, as applied to solidification and freezing—are usually based on the assumption of instantaneous contact of liquid with the cooling surface. In fact the liquid to be solidified on the walls is introduced into the channel or cavity, and a blockade of flow due to complete solidification in a certain cross-section is always possible. This problem is analysed in the present paper.

NOMENCLATURE

- a, thermal diffusivity of the solid phase;
- b, half-width of the channel;
- c_f, friction factor;
- f, dimensionless length of the casting;
- l, length of the casting;
- L, characteristic length;
- m, mass of the casting;
- Δp, pressure drop;
- R, radius;
- t, time;
- U, freezing constant;
- V, volume;
- w, velocity;
- x, co-ordinate;
- y, thickness of the solidified layer;
- z, dimensionless time.

Greek symbols

- ε, constant, equation (2.11);
- ζ, dimensionless variable;
- μ, viscosity;
- ρ, ρ', densities of the solid and liquid phases, respectively;
- τ, time;
- φ, dimensionless variable;
- ω, dimensionless velocity.

1. INTRODUCTION

THE SOLUTIONS of the Stefan problems, connected with solidification and freezing, which may be found in the literature (e.g. [1]), are based as a rule on the assumption of instantaneous contact of liquid with the cooling surface (or mould). This assumption might be justified only in such cases where the solidification process is much slower than the process of flow, and this is usually not true. As a matter of fact the liquid to be solidified freezes on the walls of the cooling surface while flowing through the channel, and a blockade of flow due to complete solidification in a certain cross-section is always possible. To analyze such a situation the solutions of the corresponding Stefan problem must be known. That is, if the walls of the channel are flat, one must have ready solution of the Stefan problem for a flat wall in which the instantaneous contact of all the surface with the solidifying liquid is assumed. For such cases the thickness of the solidified layer is given by

$$y = U\sqrt{(at')}, \tag{1.1}$$

where U is a freezing constant satisfying a certain transcendent equation, a is the thermal diffusivity of the solidified layer, and t' denotes time counted from the moment of instantaneous contact. Thus the equation (1.1) can be utilized for all two-dimensional cases with flat walls. In the case of a round tube the relation (1.1) is not true, but a similar solution exists [2]. The solution of the type (1.1) offers thus the basis for a more elaborate analysis of the solidification during the flow of liquid.

2. TWO-DIMENSIONAL CASE OF A CAVITY WITH FLAT WALLS

In Fig. 1 a sketch of the geometry to be analyzed is given. It is assumed that the angle α is sufficiently small, therefore it is approximately $b_0 = s_0$, and $b = s$.

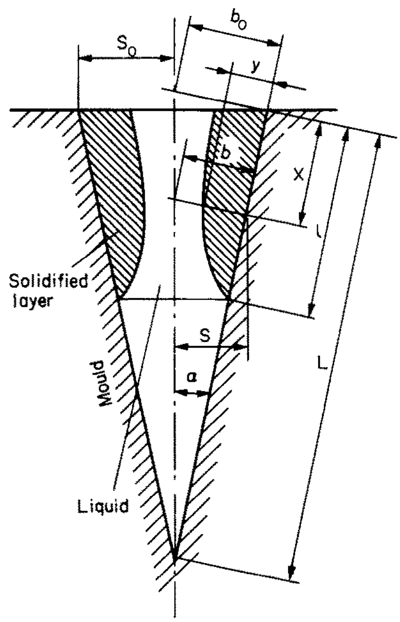

FIG. 1.

The position of the liquid front is given by $l(t) = x$. The thickness of the solidified layer $y(x, t)$ may be calculated with the help of the relationship (1.1) in which t' is replaced by

$$t' = t - \tau, \tag{2.1}$$

where t is the actual time, and τ the time at which the liquid front has reached the place x. Thus we have

$$l(\tau) = x \tag{2.2}$$

and

$$y(x, t) = U\sqrt{[a(t-\tau)]}. \tag{2.3}$$

At the entrance, $x = 0$, it is of course

$$y(0, t) = U\sqrt{(at)} = y_0. \tag{2.4}$$

Let us calculate the volume of the solidified layer taking into account one half of the cavity and unit depth of it; namely

$$V(t) = \int_0^1 y\,dx = \int_0^t \frac{dl(\tau)}{d\tau} U\sqrt{[a(t-\tau)]}\,d\tau. \tag{2.5}$$

For the considered geometry the filled volume is equal to

$$V_c(t) = \int_0^1 b(x)\,dx = b_0 l - \frac{b_0 l^2}{2L}, \tag{2.6}$$

since

$$b(x) = b_0 - b_0(x/L). \tag{2.7}$$

Let ρ denote the density of the solid, and ρ' that of liquid. The mass of substance supplied to the cavity is then

$$m = \rho V + \rho'(V_c - V) = \int_0^t w(t)\rho'(b_0 - y_0)\,dt, \tag{2.8}$$

where $w(t)$ denotes the inlet velocity of liquid.

Substituting (2.5–6) into (2.8), introducing the dimensionless variables

$$f = \frac{l}{L}, \quad \varphi = \frac{x}{L}, \quad z = \frac{aU^2 t}{b_0^2}, \quad \zeta = \frac{aU^2\tau}{b_0^2}, \tag{2.9}$$

and rearranging we arrive at an integral Volterra equation of the second kind

$$\omega(z)\cdot(1-\sqrt{z}) = f'(z)\cdot(1-f) + \varepsilon\int_0^z f'(\zeta)\frac{d\zeta}{\sqrt{(z-\zeta)}}, \tag{2.10}$$

where

$$\omega(z) = \frac{b_0^2 w(z)}{aLU^2}, \quad \varepsilon = \frac{\rho - \rho'}{2\rho'}, \tag{2.11}$$

and

$$f'(z) = \frac{df(z)}{dz}. \tag{2.12}$$

3. SPECIAL CASE OF A FLAT SLIT

In this particular case it is $L = \infty$ with regard to the sketch in Fig. 1. Therefore in the equation (2.10) the term $(1-f)$ should be replaced by 1; after this operation L cancels out in ω and f', so that one can take for L an arbitrary reference length without changing the symbols in (2.10–12). Thus we have

$$\omega(z)\cdot(1-\sqrt{z}) = f'(z) + \varepsilon\int_0^z f'(\zeta)\frac{d\zeta}{\sqrt{(z-\zeta)}}. \tag{3.1}$$

Let us consider the case of constant inlet velocity, $\omega = \text{const}$. The solution $f'(z)$ may be quite easily obtained by means of the subsequent-approximations technique, which is quickly convergent due to small values of the parameter ε. Starting from the first approximation

$$f'(z) = \omega(1-\sqrt{z}) \tag{3.2}$$

we arrive in two steps at the third approximation

$$f_3'(z) = \omega\left[1-(\sqrt{z}) - 2\varepsilon\sqrt{z}\left(1-\frac{\pi}{4}\sqrt{z}\right) + \varepsilon^2\pi z(1-\tfrac{2}{3}\sqrt{z})\right], \tag{3.3}$$

which may be used as the final result. Integration yields

$$f_1(z) = \omega z(1-\tfrac{2}{3}\sqrt{z}) \tag{3.4}$$

and

$$f_3(z) = \omega z\left[1-\tfrac{2}{3}(\sqrt{z}) - \tfrac{4}{3}\varepsilon\sqrt{z}\left(1-\frac{3\pi}{16}\sqrt{z}\right) + \frac{\pi}{2}\varepsilon^2 z(1-\tfrac{8}{15}\sqrt{z})\right]. \tag{3.5}$$

The entrance is closed by the solidified layer when $y_0 = b$, i.e. for $z = 1$. Hence

$$f_1(1) = \omega/3, \tag{3.6}$$

$$f_3(1) = \frac{\omega}{3}(1 - 1.6437\varepsilon + 2.1991\varepsilon^2). \tag{3.7}$$

The quantity ε is usually sufficiently small, e.g. for tin it is $\varepsilon = 0.02111$, whence $f_3(1) = 0.3221\omega$, which differs from the first approximation $f_1(1) = \omega/3$ by -3.5%. For water $\varepsilon = -0.0415$, hence $f_3(1) = 0.3573\omega$, and thus the error is $+6.7\%$. It is felt that in many cases the first approximation is sufficient, which means that usually $\varepsilon = 0$ may be put.

The case of constant inlet velocity is less realistic than the case of constant pressure drop Δp. However it seems very difficult to determine the latter. For the sake of simplicity let us assume an analogous routine for determination of the pressure drop as in straight channels of constant cross-section, namely

$$\Delta p = \frac{c_f l}{b_0 - y_0}\cdot\frac{\rho' w^2}{2}, \tag{3.8}$$

where c_f is the friction factor being in general a function of the Reynolds number assumed thus

$$(Re) = \frac{4\rho' w(b_0 - y_0)}{\mu}, \tag{3.9}$$

where μ is the viscosity. We will consider two limiting cases, the first of laminar flow, when

$$c_f = \frac{24}{(Re)}, \tag{3.10}$$

and the second of the highly developed turbulent flow with $c_f = \text{const}$. In the first case we have

$$\omega(z) = \omega_1 \cdot \frac{(1-\sqrt{z})^2}{f}, \quad \omega_1 = \frac{b_0^4 \Delta p}{3\mu a L^2 U^2}, \tag{3.11}$$

and in the second

$$\omega(z) = \omega_2 \sqrt{\left(\frac{1-\sqrt{z}}{f}\right)}, \quad \omega_2 = \frac{b_0^2}{aLU^2}\sqrt{\left(\frac{2b_0 \Delta p}{c_f \rho' L}\right)}. \tag{3.12}$$

The first approximations of the function $f(1)$ are given by the following formulae: for laminar flow

$$f(1) = \sqrt{(\omega_1/5)}, \tag{3.13}$$

and for turbulent flow

$$f(1) = (12\omega_2/35)^{2/3}. \tag{3.14}$$

Hence the maximum length of the casting $l_{\max}$ may be calculated. For the constant velocity case

$$l_{\max} = \frac{b_0^2 w}{3aU^2}; \tag{3.15}$$

for laminar flow

$$l_{\max} = 0.2582 \frac{b_0^2}{U}\sqrt{\left(\frac{\Delta p}{\mu a}\right)}; \tag{3.16}$$

and for turbulent flow

$$l_{\max} = 0.6172 \frac{b_0^2}{U}\left(\frac{\Delta p}{c_f a^2 b_0 U \rho'}\right)^{\frac{1}{3}}. \tag{3.17}$$

On comparing the above formulae the reference velocity for the case of constant pressure drop may be calculated thus: for laminar flow

$$w = 0.7746 U \sqrt{\left(\frac{a\Delta p}{\mu}\right)}; \tag{3.18}$$

and for turbulent flow

$$w = 1.8517\left(\frac{U^2 a \Delta p}{c_f b_0 \rho'}\right)^{\frac{1}{3}}. \tag{3.19}$$

4. FILLING OF THE CAVITIES

If the angle $\alpha = 0$ (see Fig. 1) then the blockade of flow occurs always at the entrance. This is also possible in the case of $\alpha > 0$, but sometimes the blockade may take place inside the cavity (Fig. 2b). To analyse this problem we will use only the first approximation of the solution $f(z)$, on putting $\varepsilon = 0$ in (2.10). Let us consider first the case of constant velocity $\omega = \text{const}$. Then

$$f = 1 - \sqrt{[1 - 2\omega z(1 - \tfrac{2}{3}\sqrt{z})]}. \tag{4.1}$$

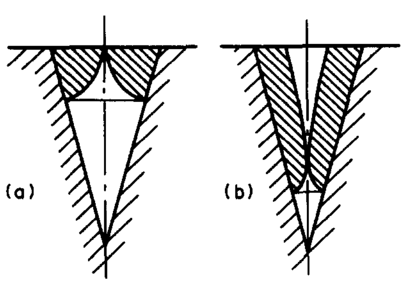

FIG. 2.

Hence

$$\varphi = 1 - \sqrt{[1 - 2\omega\zeta(1 - \tfrac{2}{3}\sqrt{\zeta})]}. \tag{4.2}$$

The condition of the blockade may be specified thus

$$y = b, \tag{4.3}$$

or

$$U\sqrt{[a(t-\tau)]} = b_0\left(1 - \frac{x}{L}\right). \tag{4.4}$$

With use of (2.9) we obtain

$$\sqrt{(z-\zeta)} = 1 - \varphi. \tag{4.5}$$

Substituting (4.2) yields

$$\sqrt{(z-\zeta)} = \sqrt{[1 - 2\omega\zeta(1 - \tfrac{2}{3}\sqrt{\zeta})]}. \tag{4.6}$$

A supplementary condition for the blockade inside the cavity is evidently

$$\frac{dy}{dx} = \frac{db}{dx} = -\frac{b_0}{L}, \tag{4.7}$$

or, with use of (2.9),

$$\frac{d}{d\varphi}\sqrt{(z-\zeta)} = -1. \tag{4.8}$$

Hence

$$\frac{d\varphi}{d\zeta} = \frac{1}{2(1-\varphi)} \tag{4.9}$$

at the place of the blockade. Substituting (4.2) and solving for ζ we obtain

$$\zeta = \left(1 - \frac{1}{2\omega}\right)^2, \tag{4.10}$$

whence

$$\varphi = 1 - \sqrt{\left[1 - \tfrac{2}{3}(\omega+1)\left(1 - \frac{1}{2\omega}\right)^2\right]}, \tag{4.11}$$

and

$$z = 1 + \tfrac{1}{3}(1-2\omega)\left(1 - \frac{1}{2\omega}\right)^2. \tag{4.12}$$

The function φ determines the place of blockade, and z the time of the event. The length of the casting $l = Lf$ is determined by (4.1) and (4.2) thus

$$f = 1 - \sqrt{\left\{1 - 2\omega + \tfrac{1}{3}(1-2\omega)^2\left(1 - \frac{1}{2\omega}\right) + \tfrac{4}{3}\omega\left[1 + \tfrac{1}{3}(1-2\omega)\left(1 - \frac{1}{2\omega}\right)^2\right]^{\frac{3}{2}}\right\}}. \tag{4.13}$$

Note that when $2\omega \leqslant 1$ the blockade occurs at the entrance, and in this case

$$\zeta = 0, \quad \varphi = 0, \quad z = 1, \quad f = 1 - \sqrt{(1 - \tfrac{4}{3}\omega)}. \tag{4.14}$$

If the cavity is properly degassed it can be filled completely if $\omega \geqslant 1.8346$. The latter value is the solution of (4.13) for $f = 1$. Thus we have the following results: blockade at the entrance for $0 < \omega < 0.5$; blockade within the cavity for $0.5 < \omega < 1.8346$; complete filling of the cavity for $\omega > 1.8346$ (proper degassing provided).

A similar analysis may be performed for the two cases of constant pressure drop. In result it has been found that the complete filling of the cavity is not possible. For laminar flow we have obtained $f_{max} = 0.8800$ at $z = 0.2441$; the place of blockade is determined by $\varphi = 0.7500$ at $\omega_1 = 1.9882$. For turbulent flow it is $f_{max} = 0.9570$, $z = 0.3466$, $\varphi = 0.8333$, $\omega_2 = 1.5886$.

5. ROUND HOLE

We will consider now a round hole (mould) of radius R into which liquid to be solidified is introduced (Fig. 3). Contrary to the preceding analysis $y(x)$ now denotes the local radius of the liquid column, and $y_0 = y(0)$. To solve the problem the solution of the

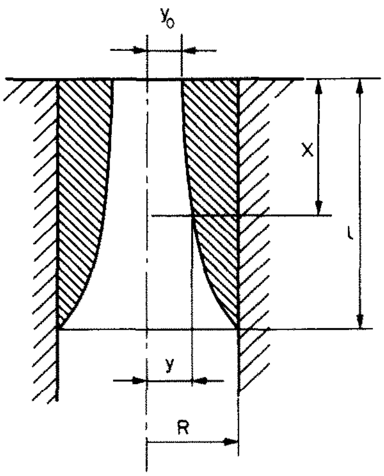

FIG. 3.

analogous Stefan problem must be known, in which liquid is introduced into the mould instantaneously, and the thickness of the solidified layer is constant, i.e. $y(x) = y_0$. The solution of this problem was given in [2] in the form

$$y_0 = R\left[1 - U\sqrt{\left(\frac{at'}{R^2}\right)} - A_1\frac{at'}{R^2} - A_2\left(\frac{at'}{R^2}\right)^{\frac{3}{2}} - \dots\right], \quad (5.1)$$

where U is calculated as for the flat mould, and the coefficients $A_1, A_2, \dots$ depend among others upon U [2]. Using the new variables

$$z = \frac{aU^2 t}{R^2}, \quad \zeta = \frac{aU^2\tau}{R^2}, \quad \alpha_i = \frac{A_i}{U^{i+1}}, \quad (5.2)$$

we make use of the function

$$F(z) = 1 - (\sqrt{z}) - \alpha_1 z - \alpha_2 z^{\frac{3}{2}} - \dots. \quad (5.3)$$

Evidently

$$y_0 = RF(z), \quad y = RF(z-\zeta). \quad (5.4)$$

We can now calculate the volume of the solidified layer

$$V(t) = \int_0^l \pi(R^2 - y^2)\,dx \quad (5.5)$$

and the mass introduced into the mould

$$m = \rho'\pi R^2 l + (\rho - \rho')V(t)$$
$$= \int_0^t w(t)\rho'\pi(R^2 - y_0^2)\,dt. \quad (5.6)$$

In result the following integral equation is obtained

$$\omega(z)\psi(z) = f'(z) + 2\varepsilon\int_0^z f'(\zeta)\psi'(z-\zeta)\,d\zeta, \quad (5.7)$$

where

$$\omega(z) = \frac{R^2 w(t)}{aLU^2}, \quad \varepsilon = \frac{\rho - \rho'}{2\rho'}, \quad (5.8)$$

and

$$f'(z) = \frac{df(z)}{dz}, \quad \psi'(z) = \frac{d\psi(z)}{dz}, \quad \psi(z) = 1 - F^2(z). \quad (5.9)$$

Let us consider the case of constant velocity ω = const with restriction to the first approximation, $\varepsilon = 0$. We have thus

$$\frac{f(z)}{\omega} = \tfrac{4}{3}z^{\frac{3}{2}} - \frac{1-2\alpha_1}{2}z^2 - \tfrac{4}{5}\alpha_1 z^{\frac{5}{2}} - \frac{\alpha_1}{3}z^3 - \dots. \quad (5.10)$$

The blockade occurs in this case only at the entrance when $y_0 = 0$ or

$$F(z) = 0 = 1 - (\sqrt{z}) - \alpha_1 z - \dots. \quad (5.11)$$

Knowing the coefficients α_i we can find the actual value of the dimensionless time z from (5.14), and hence calculate the maximum value of f from (5.10). For a special case $\alpha_1 = 0.4$, $\alpha_i = 0$ for $i \geqslant 2$ we have obtained $z = 0.5861$ and $f = 0.4529$ from the first approximation. The second approximation has been also calculated with the result

$$f = (0.4529 - 0.4754\varepsilon)\omega. \quad (5.12)$$

Thus for tin with $\varepsilon = 0.02111$ the error committed was -2.2%, and for water with $\varepsilon = -0.0415$ the error was $+4.4\%$.

The same analysis can be performed for the cases of constant pressure drop. In this case it is assumed

$$\Delta p = \frac{l}{y_0} c_f \rho' w^2 \quad (5.13)$$

with c_f = const for turbulent flow, and

$$\Delta p = 8\mu l w / y_0^2 \quad (5.14)$$

for laminar flow. In the latter case the first approximation $f(z)$ fulfils the equation

$$\frac{df}{dz} = \omega_1\psi(z)\cdot\frac{[1 - (\sqrt{z}) - \alpha_1 z - \dots]^2}{f}, \quad (5.15)$$

where

$$\omega_1 = \frac{R^2\Delta p}{8\mu a L^2 U^2}, \quad (5.16)$$

and hence $f(z)$ can be easily calculated once the coefficients α_i are known. For the case of flow of sodium in a stainless steel tube it has been found by the method given in [2] that $U = 0.5654$, $\alpha_1 = 0.5388$, and other coefficients α_i are sufficiently small to be neglected. Hence it has been obtained $z = 0.5190$ and $f = 0.3326\sqrt{(\omega_1)}$. For turbulent flow the result was $f = 0.4526\omega_2^{2/3}$ for the blockade, where

$$\omega_2 = \frac{R^2}{aLU^2}\sqrt{\left(\frac{R\Delta p}{c_f\rho' L}\right)}. \quad (5.17)$$

6. INFLUENCE OF THE FLUCTUATIONS OF FLOW

The flow of liquid may be subject to fluctuations of various origin. To study their effect the equation (3.1) will be used with $\varepsilon = 0$ (first approximation). Let us assume

$$\omega(z) = \bar{\omega}(1-\alpha \sin \nu z), \tag{6.1}$$

where α is the amplitude, and ν the frequency of the oscillations. Solving (3.1) for f we obtain

$$f(z) = \bar{\omega}\left[z(1-\tfrac{2}{3}\sqrt{z})+\frac{\alpha}{\nu}(1-\cos \nu z) - \alpha\int_0^z \sqrt{z}\sin \nu z \, dz\right]. \tag{6.2}$$

In this case the blockade occurs at the entrance, and hence $z = 1$ for the event. The maximum reduced length of the casting is therefore

$$f(1) = \bar{\omega}\left[\tfrac{1}{3}+\frac{\alpha}{\nu}\left(1-\frac{1}{2\sqrt{z}}\int_0^1 \sqrt{z}\sin \nu z \, dz\right)\right], \tag{6.3}$$

or

$$f(1) = \bar{\omega}[\tfrac{1}{3}+\alpha\chi(\nu)]. \tag{6.4}$$

The function

$$\chi(\nu) = \frac{1}{\nu}\left(1-\frac{1}{2\sqrt{\nu}}\int_0^{\nu} \frac{\cos u}{\sqrt{u}} du\right) \tag{6.5}$$

is given in Table 1. It exhibits a maximum for $\nu = ca. 3.3$, where $\chi_{max} = 0.1995$. Thus there is a certain frequency

$$\nu^* = \frac{\nu z}{2\pi t} = \frac{3.3}{2\pi}\cdot\frac{aU^2}{b_0^2}, \tag{6.6}$$

at which there is the greatest influence of the flow fluctuations. In this case

$$f(1) = \frac{l_{max}}{L} = \frac{b_0^2 \bar{w}}{3aLU^2}\cdot(1+0.5985\alpha). \tag{6.7}$$

In particular for sodium flowing in stainless steel channels we have $U = 0.5654$, $a = 6.69\times10^{-5}\,\mathrm{m^2\,s^{-1}}$, e.g. for $b_0 = 2.5\times10^{-3}$ m it is $\nu^* = 1.797\,\mathrm{s^{-1}}$.

If there are two parallel channels subject to oscillations with equal amplitudes but different phases so that $\alpha_1 = -\alpha_2 = \alpha$, then

$$\left.\begin{aligned} \frac{3aU^2 l_{max,1}}{\bar{w}b_0^2} &= 1+3\alpha\chi(\nu), \\ \frac{3aU^2 l_{max,2}}{\bar{w}b_0^2} &= 1-3\alpha\chi(\nu), \end{aligned}\right\} \tag{6.8}$$

and thus in the case of $\chi = \chi_{max} = 0.1995$ it is

$$\frac{l_{max,2}}{l_{max,1}} = \frac{1-0.5985}{1+0.5985\alpha}. \tag{6.9}$$

This characterizes the non-uniformity of liquid distribution.

Table 1

ν	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$\chi(\nu)$	0	0.0200	0.0381	0.0574	0.0764	0.0945	0.1115	0.1272
ν	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
$\chi(\nu)$	0.1416	0.1545	0.1658	0.1755	0.1835	0.1899	0.1946	0.1977
ν	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6
$\chi(\nu)$	0.1933	0.1994	0.1982	0.1957	0.1921	0.1876	0.1823	0.1763
ν	4.8	5.0	5.2	5.4	5.6	5.8	6.0	6.2
$\chi(\nu)$	0.1698	0.1630	0.1561	0.1490	0.1420	0.1353	0.1288	0.1226
ν	6.4	6.6	6.8	7.0	7.2	7.4	7.6	7.8
$\chi(\nu)$	0.1169	0.1117	0.1070	0.1029	0.0992	0.0961	0.0935	0.0913
ν	8.0	8.2	8.4	8.6	8.8	9.0	9.2	9.4
$\chi(\nu)$	0.0895	0.0881	0.0870	0.0862	0.0856	0.0850	0.0846	0.0842
ν	9.6	9.8	10.0	10.2	10.4	10.6	10.8	11.0
$\chi(\nu)$	0.0837	0.0832	0.0826	0.0819	0.0811	0.0801	0.0790	0.0778

7. CONCLUSIONS

Discussion of the model used

The problems of solidification are very often solved with the assumption of instantaneous contact of the whole cooling surface with the solidifying liquid. In fact the processes of filling and of solidification may occur at a commensurable rate. As a consequence the blockade of channels occurs. In this paper the problem is reduced to the solution of integral Volterra equations of the second kind. If the densities of the phases involved do not differ very much the first approximation is usually quite sufficient to describe the phenomenon. The following geometries have been taken into account: two-dimensional enclosure with flat walls; flat slit; and round tube. Three cases of flow have been considered, namely that of constant velocity, and those of constant pressure drop at laminar or turbulent flow.

The presented theory is based on the assumption that heat conduction in the direction of liquid flow, in the mould as well as in the solidified layer, can be neglected. If this were true, the use of one-dimensional freezing law in two-dimensional cases would be justified. In fact, if the walls of the mould are kept at constant temperature, the curvature of the isotherms in the solidified layer is the greatest near the liquid front, and namely there the basic assumption is not completely fulfilled. Contrariwise, far from the liquid front the basic assumption seems to be justified. In the case of an uncooled semi-infinite mould the situation is quite analogous. In such a mould in one-dimensional cases the temperature at the walls assumes instantaneously a constant value during the contact with the freezing liquid. Therefore the heat flux in the direction of flow at the liquid front must be expected. The influence of these phenomena is obscure at the moment; anyway the presented solution may be used as a first approximation for the whole process, and may be also the starting point for more sophisticated analysis.

In practical use of the presented theory one must be aware that the times of processes in moulds of constant cross-section are independent of the velocity of liquid supply, e.g. from Sections 2 and 3 it follows that for the first approximation in the case of a flat slit it is for the moment of the blockade $z = 1, f = \omega/3$ at constant velocity. Hence one obtains the length of the casting $l = wt/3$, where $t = (b_0/U)^2/a$ is the time of the freezing process at the inlet. Thus to obtain great lengths of the casting one must create sufficiently great supply velocities.

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PHENOMENE DE SOLIDIFICATION DANS L'ECOULEMENT EN CONDUITE ET DANS DES CAVITES

Résumé—Les solutions du problème de Stefan, appliquées à la solidification et à la congélation sont généralement basées sur une hypothèse de contact instantané du liquide avec la surface de refroidissement. En réalité le liquide à solidifier sur les parois doit être introduit dans le canal ou dans la cavité et une obstruction dûe à la solidification complète dans une section est toujours possible. Ce problème est analysé dans le présent article.

DER ERSTARRUNGSVORGANG BEI STRÖMUNGEN IN KANÄLEN UND RAUHIGKEITSVERTIEFUNGEN

Zusammenfassung—Die Lösungen des Stefan-Problems, wie sie auf Erstarrungs- und Gefrierprobleme angewandt werden, basieren gewöhnlich auf der Annahme eines sofortigen Kontaktes der Flüssigkeit mit der Kühlfläche. Tatsächlich wird die erstarrende Flüssigkeit in einen Kanal oder eine Rauhigkeitsvertiefung eingeführt und es ist immer eine Blockierung der Strömung infolge vollständiger Erstarrung in einem bestimmten Querschnitt möglich. Dieses Problem wird in der vorliegenden Arbeit untersucht.

ЗАТВЕРДЕВАНИЕ ПРИ ТЕЧЕНИИ ЧЕРЕЗ КАНАЛЫ И В ПОЛОСТЯХ

Аннотация — Решение задачи Стефана, используемые для затвердевания и замерзания, обычно основаны на предположении о мгновенном контакте жидкости с охлаждаемой поверхностью В действительности жидкость, затвердевающая на стенках, подается в канал или полость и всегда имеется возможность блокирования течения вследствие полного затвердевания в некотором поперечном сечении. Данная задача исследуется в настоящей статье.